355 – Generation of Oscillations

Acquired core competencies:

1. You recognize that the operation of typical basic op amp circuits is determined by the external components (here: R_1, R_2), and you verify this quantitatively.
2. You break down a complex circuit into its functional blocks (in this case: amplifier + resonant circuit) and thus intuitively understand how they interact.
3. You solve a "chicken-and-egg problem" regarding the generation of a stable electrical oscillation with frequency f_0.

1 Tasks

1. Determine the phase shift and the amplitude gain of an op-amp-based inverting amplifier circuit.
2. Record the amplitude and phase transfer functions of an LC resonant circuit and compare them with theoretical predictions.
3. Combine the LC resonant circuit with the inverting amplifier to form a Meissner oscillator circuit. Determine the frequency of the oscillator and its stability.

2 Fundamentals

Keywords: resonant circuit, resonance frequency, transfer function, operational amplifier, Meissner oscillator circuit (a.k.a. Armstrong oscillator)

2.1 Operational amplifier (op amp)

The op amp is an important component in modern electronic circuits. It is a so-called differential amplifier and has **two signal inputs** but only **one signal output** (see Fig. 1). The input impedance of an ideal op amp is infinite, meaning that no current flows into the op amp's two signal inputs ($I_- = I_+ = 0$).

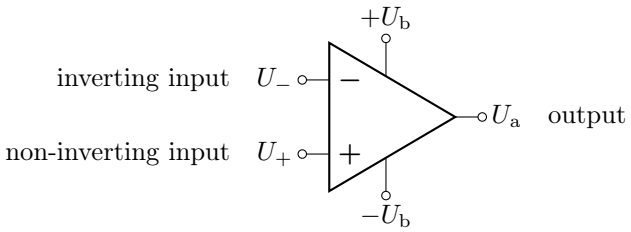


Figure 1: Typical graphical symbol for an op amp. The operating voltage (often symmetrical: $+U_\mathrm{b}$ and $-U_\mathrm{b}$) is usually omitted from the circuit diagram and tacitly assumed.

The output voltage of the *open-loop* op amp is given by:

$$U_\mathrm{a} = V_0 \cdot (U_+ - U_-) \tag{1}$$

that is, the output voltage corresponds to the **amplified difference between the input signals**. A typical operational amplifier has an open-loop voltage gain V_0 in the range from 10^5 to 10^6.

In practice, it is often impossible to work with such high gain levels, since even noise amplitudes in the µV range can then produce large output signals. Furthermore, only small signal gains (e.g., 10 or 100) are often necessary. This is achieved through what is known as **negative feedback**: a portion of the output voltage is fed back to the inverting input (see Fig. 2). As soon as the output voltage rises, more and more of it is fed back to the inverting input. This causes the input voltage difference to decrease. A feedback loop is formed – the more the output signal increases, the smaller the input signal becomes. When the feedback voltage component equals the voltage at the non-inverting input, equilibrium is reached and the **feedback loop stabilizes**. The settling time for this feedback equilibrium is very short ⇒ frequencies up to the high kHz range or even the MHz range can be processed. In the steady-state condition, $U_+ - U_- \to 0$ is reached. Note that this does not mean that the output voltage must also be zero. Equation (1) applies only to the *open-loop* op amp!

In practice, there are essentially two ways to implement negative feedback: the inverting and the non-inverting amplifier (see Fig. 2). The voltage gain of these circuits can be easily calculated (see below).

(a) non-inverting amplifier (b) inverting amplifier

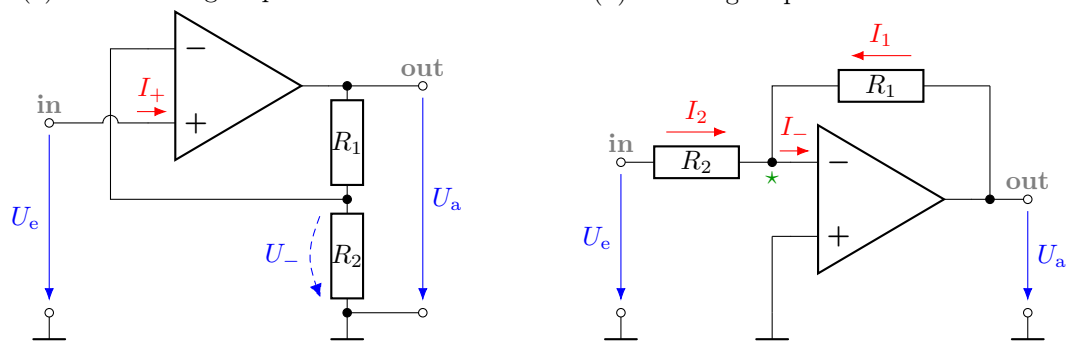


Figure 2: The two basic circuits of op amps with negative feedback. In the inverting amplifier, node ⋆ is at "virtual ground". Furthermore, $I_+ \approx I_- \approx 0$ is fulfilled.

In the **non-inverting amplifier** shown in Fig. 2(a), the signal to be fed back is generated by a voltage divider at the output. This results in the following voltage at the inverting input of the op amp (☞ voltage divider rule, Experiment 350):

$$U_- = U_a \cdot \frac{R_2}{R_1 + R_2} \tag{2}$$

In the steady-state condition, $U_+ - U_- \to 0$ is fulfilled. Since the input voltage is applied *directly* to the non-inverting input, $U_+ = U_e$ holds in this case (furthermore, the input resistance of the non-inverting amplifier is equal to that of the op amp, i.e., extremely high). This yields the following relationship between the output and input voltages: $V = U_a/U_e$:

$$\boxed{\textbf{non-inverting amplifier:} \qquad V = 1 + \frac{R_1}{R_2}} \qquad \text{with} \qquad R_{in} \to \infty \tag{3}$$

Equation (3) states that the overall gain of the circuit is determined solely by the ratio of the resistors in the voltage divider; the specific parameters of the op amp are irrelevant.

For the **inverting amplifier** in Fig. 2(b), we balance the currents at the inverting input of the op amp. In the steady-state condition, $U_+ - U_- \to 0$ must hold again. Since the non-inverting input is connected to ground here ($U_+ = 0\,\text{V}$), the inverting input must be at the same potential ($U_- \approx 0\,\text{V}$). Because the inverting input is artificially held at ground in this circuit, this point is also referred to as virtual ground. All currents at node ⋆ in Fig. 2(b) follow the junction rule: the sum of the incoming currents equals the sum of the outgoing currents. The op amp has a very high input impedance, so no current flows into the inverting input.

Therefore, the two currents I_1 and I_2 must add up to zero (however, the input current I_2 of the inverting amplifier is *not* zero, so its input impedance is *finite*). We can summarize:

$$I_1 = \frac{U_\mathrm{a}}{R_1} \quad , \qquad I_2 = \frac{U_\mathrm{e}}{R_2} \quad , \qquad I_1 + I_2 = 0 \tag{4}$$

Thus, the voltage gain $V = U_\mathrm{a}/U_\mathrm{e}$ is given by:

$$\boxed{\textbf{inverting amplifier:} \qquad V = -\frac{R_1}{R_2}} \qquad \text{with} \qquad R_\mathrm{in} = R_2 \tag{5}$$

Here, too, the overall gain is determined solely by the ratio of the resistors. In addition, the negative sign results in a phase shift of π or 180° between the input and output signals.

⚠ In equations (3) and (5), the voltage gain V of the op amp with feedback loop should not be confused with the voltage gain V_0 of the open-loop op amp from equation (1)!

The advantage of the op amp is that the overall function is determined solely by the **external circuitry**. Many functions can be implemented, including even analog arithmetic operations.

2.2 Resonant circuit

If an inductor L and a capacitor C are connected in parallel, a parallel resonant circuit is formed. Energy can be exchanged periodically between the capacitor and the inductor. A fully charged capacitor discharges its electric field energy when the circuit is closed. The flowing current causes a magnetic field to build up in the coil. Once the capacitor has completely discharged, all of the system's energy is stored in the coil's magnetic field. As the magnetic field decays, a current is generated that is opposite in direction to the original current (Faraday's law of induction, Lenz's law). The electric field in the capacitor builds up again, and the process can start over. If the entire process takes place without any losses (in particular, no ohmic resistance), the amplitude of the oscillation remains constant.

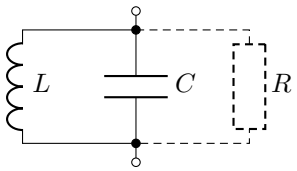


Figure 3: A parallel resonant circuit consisting of an inductor L, a capacitor C, and a resistor R. The latter usually symbolizes unavoidable losses in a practical LC circuit.

Figure 3 shows a parallel resonant circuit consisting of a coil with inductance L, a capacitor with capacitance C, and a resistive element R, which represents all losses (ohmic losses, losses in the core material). The resonant circuit has a characteristic frequency at which the reactive components of the inductor and the capacitor cancel each other out. This frequency is called the resonance frequency f_0 of the system, and it is obtained as follows:

$$i\omega_0 C + \frac{1}{i\omega_0 L} = i\left(\omega_0 C - \frac{1}{\omega_0 L}\right) = 0 \tag{6}$$

$$\Rightarrow \qquad \omega_0 = \frac{1}{\sqrt{LC}} \quad \Leftrightarrow \quad f_0 = \frac{1}{2\pi\sqrt{LC}} \tag{7}$$

⚠ For practical components, L_p and C_p must be used for the parallel resonant circuit! At resonance, the impedance of the parallel resonant circuit is at its maximum. 👉 Why?

If an idealized parallel LC resonant circuit (i.e., without the dashed resistor from Fig. 3) is supplied by a voltage source and a series resistor, a frequency-dependent voltage divider is formed, see Fig. 4. The voltage U_a varies depending on the frequency, reaching the maximum

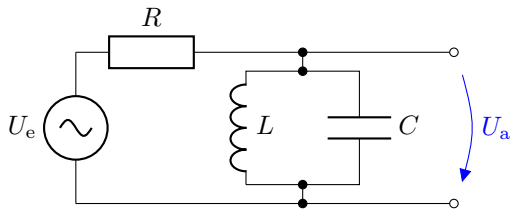


Figure 4: An idealized LC parallel resonant circuit as part of a voltage divider.

value at resonance, since the impedance of the resonant circuit is at its highest in this case. The voltage can be calculated using the ☞ voltage divider rule (see eq. (5) in Experiment 350):

$$U_a = U_e \cdot \frac{Z_C \parallel Z_L}{R + Z_C \parallel Z_L} \tag{8}$$

After a few rearrangements, the transfer function is obtained (*don't memorize this by heart!*):

$$\frac{U_a}{U_e} = \underbrace{\frac{1}{\sqrt{1 + \left(\frac{RC}{L}\right)^2 \cdot \left(\frac{1}{\omega C} - \omega L\right)^2}}}_{|U_a/U_e|} \cdot \exp\left\{ i \cdot \underbrace{\arctan\left[\left(\frac{RC}{L}\right) \cdot \left(\frac{1}{\omega C} - \omega L\right)\right]}_{\varphi} \right\} \tag{9}$$

The corresponding responses for amplitude $|U_a/U_e|$ and phase φ are plotted in Fig. 5.

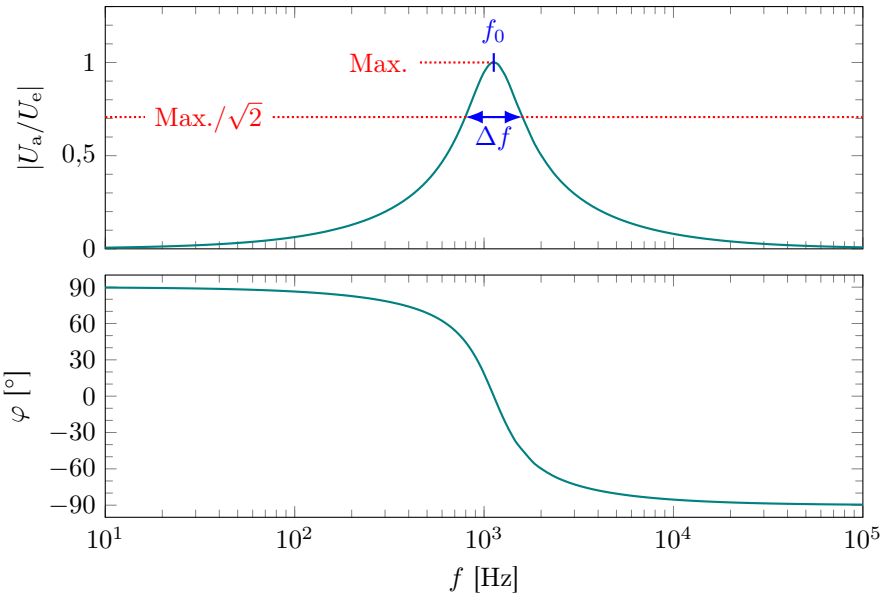


Figure 5: Amplitude response (top) and phase response (bottom) of the circuit shown in Fig. 4 for $R = 1\,\mathrm{k\Omega}$, $C = 200\,\mathrm{nF}$, $L = 100\,\mathrm{mH}$, see equation (9). $|U_a|$ is at its maximum in the case of resonance. Signals at (or near) the resonance frequency pass through the voltage divider easily, while signals at other frequencies are strongly attenuated. The circuit shown in Fig. 4 is a filter that allows only signal components within a narrow frequency band to pass through, which is also known as bandpass.

The quality factor of the resonant circuit is defined as:

$$Q = \frac{f_0}{\Delta f} = \frac{\omega_0}{\Delta\omega} = \frac{\sqrt{\frac{1}{LC}}}{\frac{1}{RC}} = R \cdot \sqrt{\frac{C}{L}} \tag{10}$$

Δf represents the bandwidth of the system (the frequency interval between the points at which the amplitude has dropped to $1/\sqrt{2}$ of the maximum value). f_0 is the resonance frequency of the resonant circuit.

2.3 Meissner oscillator circuit

An oscillator circuit always contains components that cause losses – for example, the coil's wire resistance, stray losses in the coil, or dielectric losses in the capacitor's dielectric. As a result, once an oscillation is initiated, it quickly comes to a halt. In many cases, however, it is desirable to be able to extract a sustained oscillation from a circuit, for example, as a frequency generator or a time standard. Similar to a swing, energy must always be supplied to the oscillator circuit at the right moment so that the oscillation is maintained. This energy can be supplied using an amplifier (see Fig. 6). To obtain a stable AC voltage at the output, the feedback signal must ensure that

a) at least the lost energy is re-supplied,

b) the energy is re-supplied with the correct phase (integer multiples of 2π).

Mathematically, this can be expressed using Barkhausen's oscillation conditions:

$$\text{a)} \quad |k| \cdot |V| \geq 1 \qquad \text{(Amplitudenbedingung)} \tag{11}$$

$$\text{b)} \quad \varphi_k - \varphi_V = n \cdot 2\pi \qquad \text{(Phasenbedingung, } n \in \mathbb{Z}\text{)} \tag{12}$$

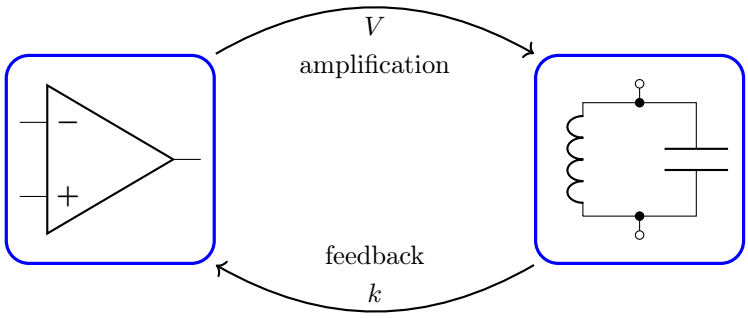


Figure 6: Generating an oscillation using an amplifier with positive feedback.

To generate a stable oscillation using a resonant circuit and an amplifier, it is necessary to couple out a portion of the resonant circuit's energy and feed it back to the amplifier in phase. One possible example is the so-called Meissner oscillator circuit shown in Fig. 9 (English speakers call it the "Armstrong oscillator"). The resonant circuit is tapped via a secondary coil using the transformer principle. The voltage induced in the secondary coil is proportional to the voltage of the resonant circuit. This voltage is then fed directly to the input of the inverting amplifier (i.e., the terminal to the left of R_2 in Fig. 2(b)). If the correct polarity of the secondary coil is selected, the feedback signal is phase-shifted by 180° relative to the resonant circuit. The inverting amplifier adds another 180°, thereby ensuring the necessary phase condition (12). Using a variable gain setting, it is now possible to feed back enough signal to compensate for the losses in the whole oscillator circuit ⇒ a stable oscillation is generated.

3 Experimental procedure

1. Set up an inverting amplifier circuit as shown in Fig. 7 and measure its overall gain $|V| = |U_a|/|U_e|$. To do this, apply a sinusoidal AC voltage (amplitude: 100 mV, frequency: 1000 Hz) to the input and determine the output amplitude. Monitor the input and output voltages using an oscilloscope in two-channel mode. If signal distortion appears at the output, you must reduce the input amplitude. Within what range (V_{min}, V_{max}) can the circuit's gain be adjusted? Compare this with the theoretical predictions, see equation (5). What is the phase relationship between the input and output signals?

   ⚠ Once you have completed this task, leave the inverting amplifier assembled. You will need this exact assembly again for task 3.

   ⚠ Leave enough contiguous space on the breadboard so that you can later arrange all the remaining components as shown in Fig. 9 without having to reconfigure the connections too much.

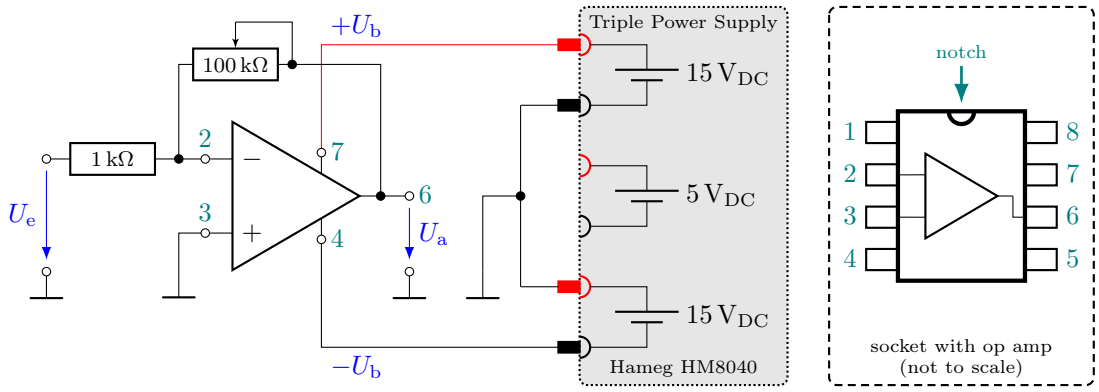


Figure 7: Inverting amplifier circuit with a symmetrical operating voltage of ±15 V, which you can obtain from the Hameg DC triple power supply. A variable 100 kΩ resistor is used in the negative feedback loop. The numbers on the op amp symbol are also printed on the socket. Pay attention to the notch located near pin 1.

2. Record the amplitude transfer function $|g(f)|$ and the phase transfer function $\varphi(f)$ of a parallel resonant circuit using LabVIEW, see Fig. 8.

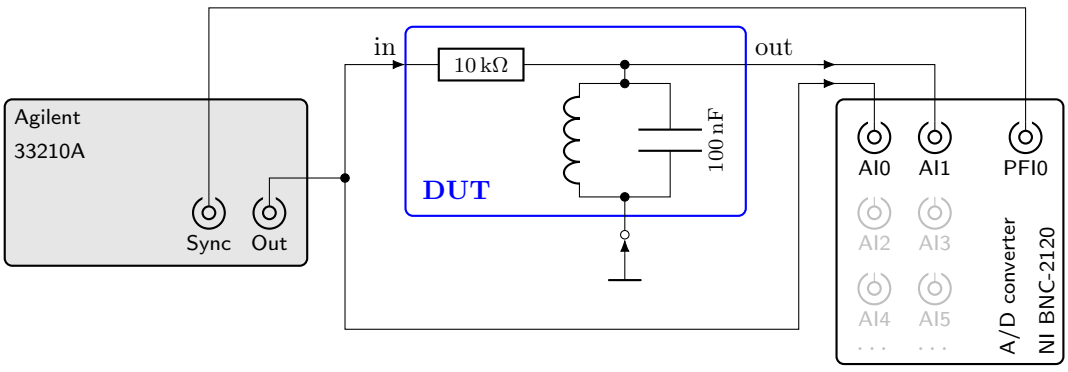


Figure 8: Flowchart for automatically recording $|g(f)|$ and $\varphi(f)$ using the LabVIEW program "Übertragungsfunktion.vi". Both the input voltage and the output voltage of the DUT (device under test) must be referenced to ground (not explicitly shown here). The FS/GS toggle switches below the AI 0 and AI 1 inputs must be set to **GS**, and the BNC/Thermocouple toggle switches above these inputs must be set to **BNC**.

⚠ Use a transformer coil with a turns ratio of 1:1 as the inductor, even though the second coil is not shown in Fig. 8 for the sake of lucidity.

⚠ Determine the resonance frequency f_0 and the quality factor Q of your resonant circuit from the data.

⚠ Leave the resonant circuit assembled after completing this task. You will need it hereafter.

3. Set up the Meissner oscillator circuit shown in Fig. 9.

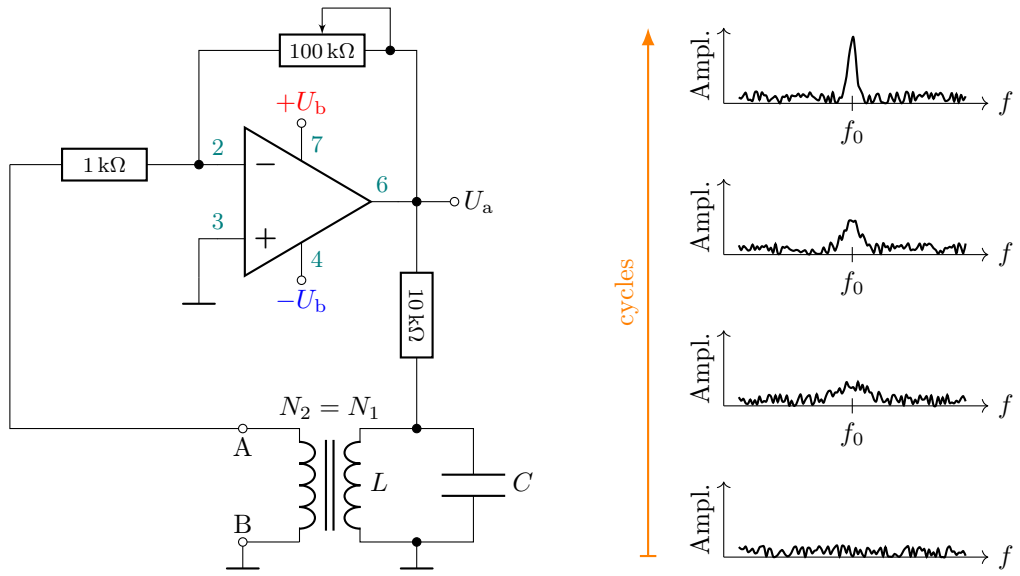


Figure 9: **(Left)** Setup of the Meissner oscillator circuit. Use the same parallel LC resonant circuit as in task 2 consisting of the transformer and the capacitor with $C = 100\,\text{nF}$. **(Right)** Generation of the oscillation via the feedback loop (schematic). Due to the ever-present background noise, there are uncorrelated signal amplitudes across a very wide frequency range ("white noise"). Frequencies near the resonance frequency of the resonant circuit are attenuated to a lesser extent. With sufficiently high gain, a signal emerges after a few cycles through the loop, since during the initial oscillation, the gain provided by the inverting amplifier is set slightly higher than what would be needed to compensate for the losses.

a) Adjust the gain until a stable oscillation is present at the output U_a of the operational amplifier. If this is not possible, you must further increase the circuit's gain. Discuss possible circuit modifications with the lab instructors. Determine the gain required to establish a stable oscillation. Record the oscillating output signal using an oscilloscope.

b) Swap the terminals A and B of the outcoupling coil N_2 on the transformer, and then swap them back again. Describe your observations.

c) Measure the oscillator's frequency and compare it with the theoretical values (by measuring the components' parameters). Compare the values obtained with the amplitude transfer function of the resonant circuit, see task 2.

d) Monitor the oscillator's frequency over a period of 2 minutes and specify the range of variation in the output signal's frequency.

e) Reduce the symmetrical operating voltage for the op amp to $\pm 10\,\text{V}$. Now connect a small loudspeaker (that will be provided to you) to the output of the op amp. Vary the parameters of the resonant circuit, for example, by adding another capacitor. Describe the sound you hear and measure the corresponding frequencies.