352 – Transfer Functions

Acquired core competencies:

1. You measure variables with a very wide range of values and make optimal use of the dynamic range of the measuring equipment.
2. You identify "regions of interest" (ROI) and adjust the measurement parameters in those regions to suit the specific conditions.
3. You fit a nonlinear function to your measurement data and compare the extracted values with direct measurements and with simple interpolations.

1 Tasks

1. Set up an RC low-pass filter *or* an RC high-pass filter.
   a) Measure the amplitude transfer function and the phase transfer function.
   b) Determine the cutoff frequency.
2. Examine a transformer with an iron core, the ratio between the numbers of turns of both coils being 1:10.
   a) Measure the amplitude transfer function and the phase transfer function.
   b) Determine the resonance frequency by amplitude and phase measurements.

2 Fundamentals

Keywords: AC circuit, transfer function, measurement of amplitude and phase, cutoff frequency, oscilloscope, electrical resistance, capacitance, inductance, transformer

2.1 AC circuit

An alternating voltage is characterized by its amplitude U_A, its frequency f, and an initial phase shift φ_0:

$$U(t) = U_\mathrm{A} \cdot \sin(2\pi f t + \varphi_0) \tag{1}$$

In many cases, an equivalent representation of this quantity is given in the complex plane:

$$U(t) = U_\mathrm{A} \cdot \exp\left(i[2\pi f t + \varphi_0]\right) \tag{2}$$

Voltage and current are related by Ohm's law:

$$Z = \frac{U}{I} \tag{3}$$

where $Z = R + iX$ represents the complex impedance. The real part of the impedance is called the resistive component R, and the imaginary part is called the reactive component X. The impedances for the ohmic resistance R, the capacitance C, and the inductance L are as follows:

$$Z_R = R \tag{4}$$

$$Z_C = \frac{1}{i\omega C} = -i\frac{1}{\omega C} \tag{5}$$

$$Z_L = +i\omega L \tag{6}$$

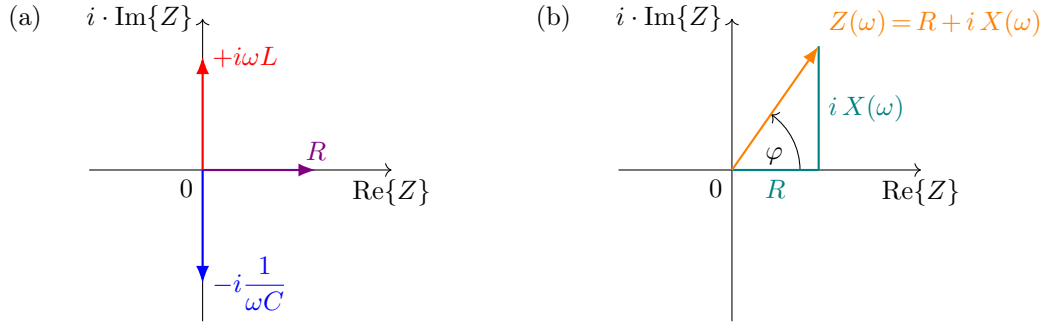


Figure 1: (a) Impedances of an ohmic resistor R, a capacitor C, and an inductor L in the complex plane. (b) Impedance $Z(\omega) = R + iX(\omega)$ of any component or assembly, e.g., for a series connection of an ohmic resistor and an inductor at a given frequency.

It can be seen that inductive and capacitive impedances are frequency-dependent and purely imaginary, whereas ohmic resistance is purely real and frequency-independent. In the complex plane, these three impedances thus exhibit the behavior shown in Fig. 1(a).

All laws applicable to DC circuits can still be applied to AC circuits if complex voltages, currents, and resistances (more precisely: impedances) are allowed. The composition of the impedance Z from its real and imaginary parts – or, alternatively, from its magnitude and phase – is given by the following relationships (see Fig. 1(b)):

$$Z = \mathrm{Re}\{Z\} + i \cdot \mathrm{Im}\{Z\} = R + iX \tag{7}$$

$$|Z| = \sqrt{[\mathrm{Re}\{Z\}]^2 + [\mathrm{Im}\{Z\}]^2} \tag{8}$$

$$\arg\{Z\} = \varphi = \arctan\left(\frac{\mathrm{Im}\{Z\}}{\mathrm{Re}\{Z\}}\right) \tag{9}$$

This results in an expression equivalent to equation (7):

$$Z = |Z| \cdot e^{i\varphi} \tag{10}$$

The amplitude, phase, and frequency of an alternating voltage can be measured using an oscilloscope. In principle, there are two different methods available for measuring phase differences. First, the phase shift $\Delta\varphi$ between the input and output signals can be determined using Lissajous figures (see Fig. 2(a)). After **zero-point calibration**, $\Delta\varphi$ can be determined by reading the axis intercepts as follows:

$$\Delta\varphi = \arcsin\left(\frac{u_x}{U_x}\right) = \arcsin\left(\frac{u_y}{U_y}\right) \tag{11}$$

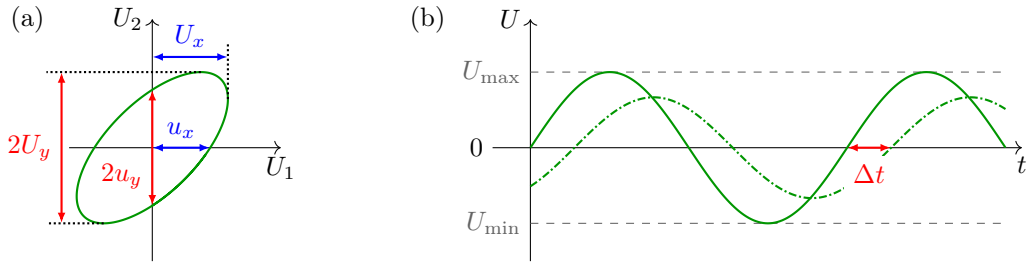


Figure 2: Phase measurement using the HM1508 two-channel oscilloscope (a) via Lissajous figures in "XY" mode and (b) via time differences in "DUAL" mode. ☞ Why is it useful to measure the *full* elongations $2u$ and $2U$ in the Lissajous figures?

Another option is to directly measure the time difference between the two signals (see Fig. 2(b)). Given the period T (corresponding to a $\Delta\varphi$ of 2π or $360°$), this time difference Δt can be converted into the phase difference $\Delta\varphi$:

$$\Delta\varphi = 360° \cdot \frac{\Delta t}{T} \mathrel{\widehat{=}} 2\pi \cdot \frac{\Delta t}{T} = \omega \cdot \Delta t \tag{12}$$

2.2 Transfer functions

In many cases, an electronic circuit has an input signal and an output signal. The (complex) relationship between these two quantities is called the transfer function:

$$g(\omega) = \frac{U_\mathrm{a}}{U_\mathrm{e}} = \frac{\text{“output”}}{\text{“input”}} \tag{13}$$

Knowing $g(\omega)$ allows an output voltage to be assigned to any given input voltage. $g(\omega)$ thus contains all the information about the assembly. In general, the transfer function is a complex quantity and can therefore also be expressed as follows:

$$g(\omega) = |g(\omega)| \cdot \exp\left[i \cdot \varphi(\omega)\right] \tag{14}$$

The quantity $|g(\omega)|$ is called the amplitude transfer function, and the phase component is called the phase transfer function $\varphi(\omega) = \arg\{g(\omega)\}$. Experimentally, the complex transfer function is determined by separately measuring the amplitude and phase transfer functions. To do this, an AC voltage of known amplitude and frequency is applied, while the amplitude and the phase difference of the output voltage are measured. By varying the frequency, we obtain the complex transfer function $g(\omega)$.

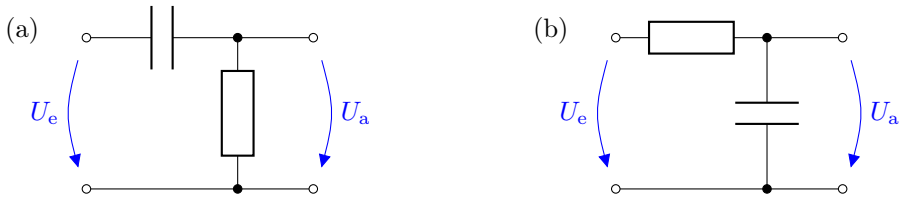


Figure 3: An RC circuit as (a) a high-pass filter or (b) a low-pass filter. In both cases, a resistive element R is connected in series with a capacitor of capacitance C. This forms a voltage divider for alternating voltage. Since the capacitor has a frequency-dependent reactance, the ratio between Z_R and Z_C is also frequency-dependent.

Figure 3 shows two configurations of RC circuits as either high-pass or low-pass filters. The corresponding transfer function of the **RC low-pass filter** is calculated using the voltage divider rule (as in Equation (5) in Experiment 350, but here with complex impedances Z):

$$U_\mathrm{a} = U_\mathrm{e} \cdot \frac{Z_C}{Z_R + Z_C} = U_\mathrm{e} \cdot \frac{\frac{1}{i\omega C}}{R + \frac{1}{i\omega C}} = U_\mathrm{e} \cdot \frac{1}{1 + i\omega RC} \tag{15}$$

$$g(\omega) = \frac{U_\mathrm{a}}{U_\mathrm{e}} = \frac{1}{1 + i\omega RC} \tag{16}$$

Let us express $g(\omega)$ in Cartesian form ($z = x + iy$):

$$\frac{1}{1 + i\omega RC} = \frac{1}{1 + i\omega RC} \cdot \underbrace{\frac{1 - i\omega RC}{1 - i\omega RC}}_{=1}$$

$$\Rightarrow \quad g(\omega) = \underbrace{\frac{1}{1 + \omega^2 R^2 C^2}}_{=\mathrm{Re}\{g\}} + i \cdot \underbrace{\frac{-\omega RC}{1 + \omega^2 R^2 C^2}}_{=\mathrm{Im}\{g\}} \tag{17}$$

The exponential form ($g = |g| \cdot e^{i\varphi}$) is derived as follows:

$$|g(\omega)| = \sqrt{[\mathrm{Re}\{g\}]^2 + [\mathrm{Im}\{g\}]^2} = \sqrt{\frac{1^2 + (-\omega RC)^2}{(1+\omega^2 R^2 C^2)^2}} = \frac{1}{\sqrt{1+\omega^2 R^2 C^2}} \tag{18}$$

$$\tan\varphi(\omega) = \frac{\mathrm{Im}\{g\}}{\mathrm{Re}\{g\}} = -\omega RC \qquad \Leftrightarrow \qquad \varphi(\omega) = \arctan(-\omega RC) \tag{19}$$

$$\Rightarrow \quad g(\omega) = \frac{1}{\sqrt{1+\omega^2 R^2 C^2}} \cdot e^{-i\arctan(\omega RC)} \tag{20}$$

The **preferred representation** of the amplitude response and the phase response covers a **wide frequency range** of approximately four orders of magnitude ("decades") in each case, compare Fig. 4.

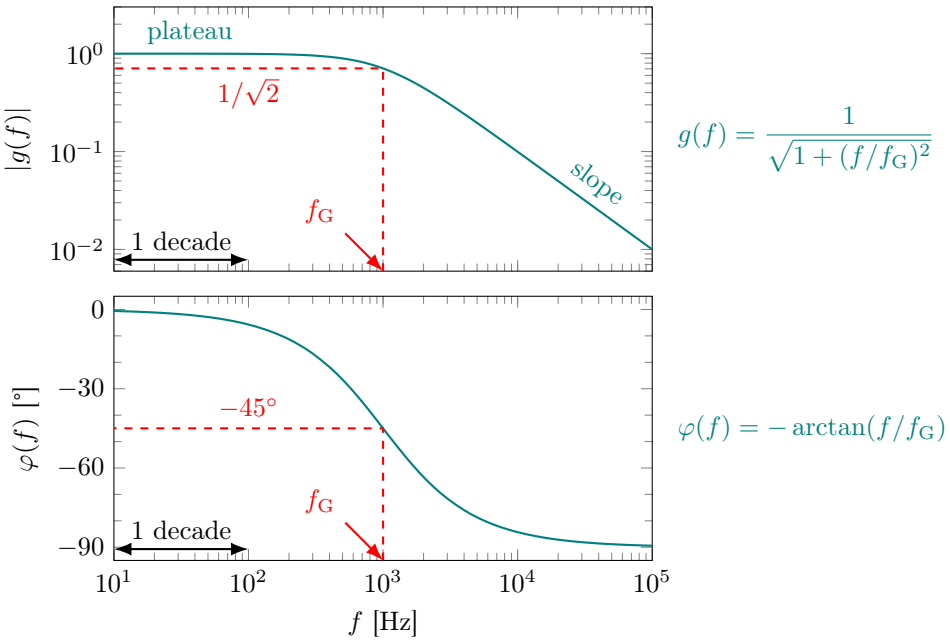


Figure 4: Amplitude response $|g(f)|$ and phase response $|\varphi(f)|$ of an RC low-pass filter plotted against the same logarithmically scaled frequency axis. The cutoff frequency of the filter used here is $f_G = 1000\,\mathrm{Hz}$. The lower graph is on a single logarithmic scale, while the upper graph is on a double logarithmic scale.

The cutoff frequency (or critical frequency) is the frequency at which the amplitude transfer function has dropped to $(1/\sqrt{2})$ times its maximum value.

For the RC low-pass filter, equation (18) thus yields a cutoff frequency of:

$$\omega_G = \frac{1}{RC} \qquad \Leftrightarrow \qquad f_G = \frac{1}{2\pi RC} \tag{21}$$

Similarly, the following transfer functions result for an **RC high-pass filter**:

$$|g(\omega)| = \frac{1}{\sqrt{1+1/[\omega^2 R^2 C^2]}} \tag{22}$$

$$\varphi(\omega) = +\arctan(1/[\omega RC]) \tag{23}$$

☞ Work through the calculation for the RC high-pass filter. Include in your report the derivation of the equations (22, 23) based on the complex voltage divider rule.

2.3 Transformer

Two coils that are magnetically coupled to each other are referred to as a transformer (see Fig. 5). They are used in many applications, such as for impedance matching, for decoupling from ground potential, or for current or voltage transformation.

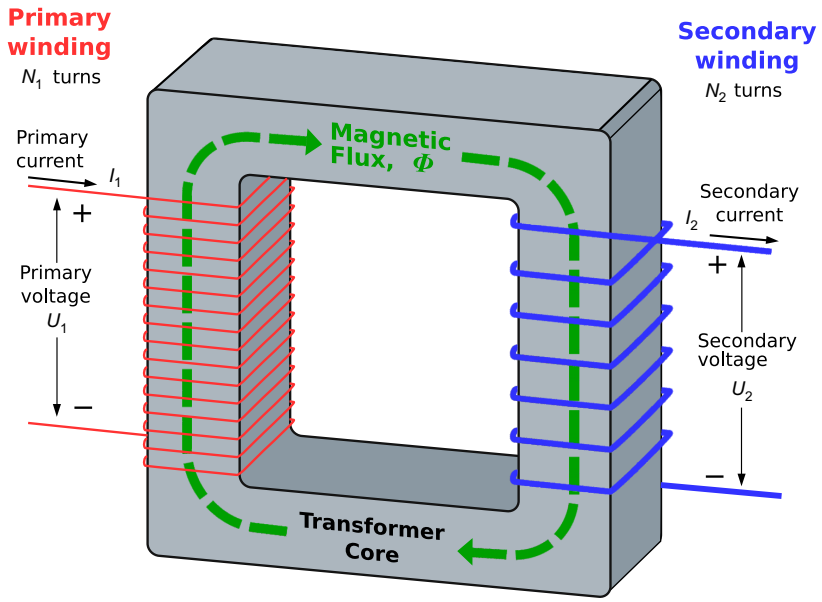


Figure 5: Scheme of a transformer consisting of two coils and a closed iron core. The primary side (index 1) and the secondary side (index 2) are each characterized by the number of turns N_n, the current I_n, and the voltage U_n. A magnetic flux of $\Phi = B \cdot F$ flows through the iron core with cross-sectional area F. Source: BillC, Omegatron, Herbertweidner, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=4813105 (Last accessed: 28.09.2026).

Assuming an ideal transformer (efficiency $\eta = 1$, neglecting stray fields and internal losses, secondary-side load resistance $R_\mathrm{L} \to \infty$), the following holds:

$$U_1 = N_1 \cdot \frac{\mathrm{d}\Phi}{\mathrm{d}t} \qquad \textit{and} \qquad U_2 = N_2 \cdot \frac{\mathrm{d}\Phi}{\mathrm{d}t} \tag{24}$$

If we consider that the magnetic flux is always the same in both coils, we immediately obtain a relationship between the number of turns N_1 and N_2 and the transfer coefficient k:

$$k = \frac{U_2}{U_1} = \frac{N_2}{N_1} \tag{25}$$

When using a magnetic material as the core for the transformer, the hysteresis behavior of the material must be taken into account (see Fig. 6). The primary current I_1 generates a magnetic flux density B in the primary coil of length l_1 according to

$$B = \mu_\mathrm{eff}\mu_0 H = \mu_\mathrm{eff}\mu_0 \frac{N_1 I_1}{l_1} \tag{26}$$

Equation (26) suggests a linear relationship between B and H. However, this is *not* the case, as shown in Fig. 6. Since $\mu_0 = 1{,}256637 \cdot 10^{-6}\,\mathrm{V\,s\,A^{-1}\,m^{-1}}$ is the magnetic field constant, it becomes clear that μ_eff is not a constant, but rather a highly nonlinear function of the magnetic field strength H (in μ_eff, both the relative permeability of the core material μ_r and the influence of any air gap are combined). When the saturation value B_max is reached, $\mu_\mathrm{eff} \to 1$. Consequently, the saturated core material is virtually indistinguishable from air; that is, its

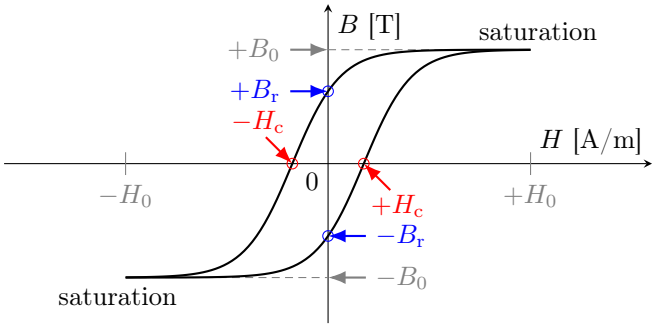


Figure 6: Hysteresis curve of a practical transformer (schematic). Symbols used: $B \mathrel{\widehat{=}}$ magnetic flux density (saturation at B_0), $H \mathrel{\widehat{=}}$ magnetic field strength (saturation at H_0), $B_\mathrm{r} \mathrel{\widehat{=}}$ remanence flux density, $H_\mathrm{c} \mathrel{\widehat{=}}$ coercive field strength. The initial magnetization curve is not shown here.

concentrating effect is increasingly lost. Thus, the secondary circuit is barely coupled to the primary circuit, which is undesirable. Saturation of the magnetic core material must therefore be avoided. ☞ Consequently, what needs to be considered in practice?

To examine the frequency dependence of the transfer function, we consider Faraday's law. If we assume that the induction is given by $\Phi = \Phi_\mathrm{max} \cdot \sin(\omega t)$, we obtain:

$$U_1 = N_1 \cdot \frac{\mathrm{d}\Phi}{\mathrm{d}t} = N_1 \cdot \Phi_\mathrm{max} \cdot \omega \cdot \cos(\omega t) = U_{1,\mathrm{max}} \cdot \cos(\omega t) \tag{27}$$

and therefore

$$U_{1,\mathrm{max}} = N_1 \cdot \Phi_\mathrm{max} \cdot \omega = N_1 \cdot B_\mathrm{max} \cdot F \cdot 2\pi f \tag{28}$$

It can be seen that, as the frequency approaches very low values, the maximum allowable voltage amplitude decreases linearly with frequency. B_max is limited by the saturation value of the magnetic flux density in the hysteresis curve.

A typical amplitude transfer function of a transformer is shown in Fig. 7. In region I, the frequencies are so low that the transformer core becomes saturated. Region II corresponds to the operating range of a transformer. Here, the input and output voltages are coupled via the turns ratio according to equation (25). In region III, resonance occurs due to the parasitic capacitance C between the turns of the coils. Together with the inductance L of the coils, this results in a resonance point at

$$f_\mathrm{R} = \frac{1}{2\pi\sqrt{LC}} \tag{29}$$

From the width of the resonance peak $\Delta f = f_\mathrm{R,o} - f_\mathrm{R,u}$ (defined as the difference between the frequencies at which the amplitude transfer function has dropped to $1/\sqrt{2}$ of the peak maximum) and the resonance frequency f_R, the quality factor Q is defined as a measure of the sharpness of a resonance:

$$Q = \frac{f_\mathrm{R}}{\Delta f} = \frac{f_\mathrm{R}}{f_\mathrm{R,o} - f_\mathrm{R,u}} \tag{30}$$

A further increase in frequency results in a decrease of the amplitude transfer function. This is caused by the reduction of the capacitive impedance $|Z_C| = 1/(\omega C)$ between the turns of the coils, as well as by eddy current losses, hysteresis losses during the remagnetization process, and possibly other effects.

The **bandwidth** of the transfer function $W = f_\mathrm{G,o} - f_\mathrm{G,u}$ is defined as the range of frequencies at which the amplitude transfer function drops to $1/\sqrt{2}$ of the plateau value $|k_\mathrm{II}|$ and **should not be confused with the width of the resonance peak**.

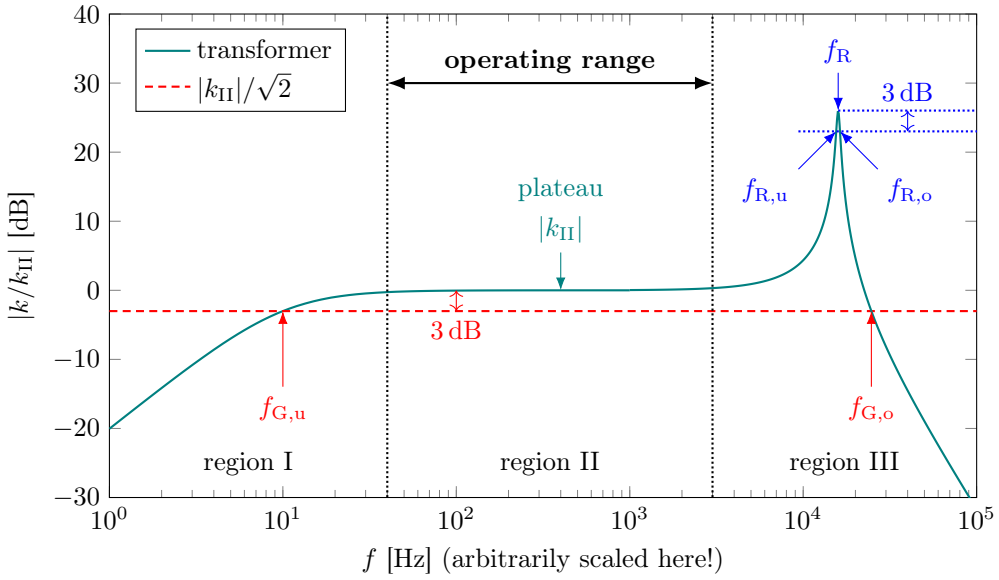


Figure 7: Ratio of output voltage to input voltage k of a transformer (schematic). The values have been normalized to the plateau (II). Note the logarithmic scale of the vertical axis, which is based on $20 \cdot \log |k/k_{II}|$ dB.

3 Experimental procedure

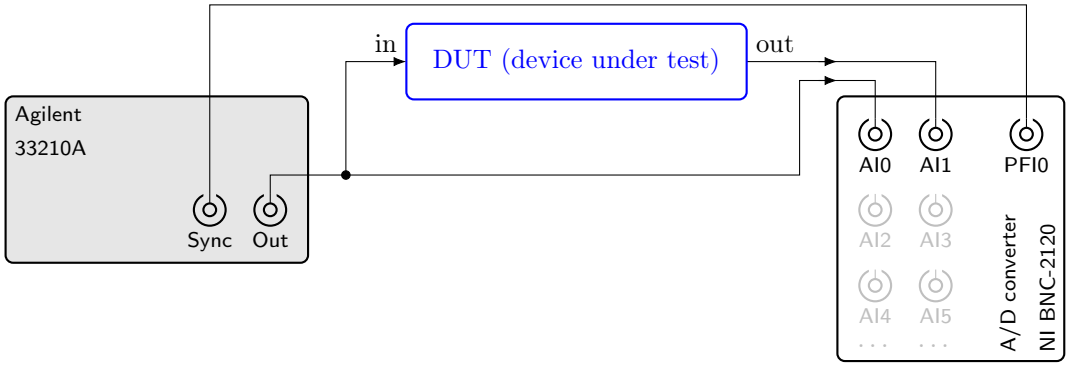


Figure 8: Flowchart for automatically recording $|g(f)|$ and $\varphi(f)$ using the LabVIEW program "Übertragungsfunktion.vi". Both the input voltage and the output voltage of the DUT (device under test) must be referenced to ground (not explicitly shown here). The FS/GS toggle switches below the AI 0 and AI 1 inputs must be set to **GS**, and the BNC/Thermocouple toggle switches above these inputs must be set to **BNC**.

Always check the waveform of the input *and* output voltage to ensure that your measurements are **always sinusoidal**! If distortion occurs, please **reduce the input voltage**.
☞ Why is this necessary? Include a brief discussion in your report!

1. Set up an RC low-pass filter *or* an RC high-pass filter (see Fig. 3).

   a) Use LabVIEW to automatically record the transfer function, see Fig. 8. Select a wide frequency range from approximately $f = 10\,\text{Hz}$ to 100 kHz and a sufficient number of measurement points that are evenly spaced on a *logarithmically* scaled frequency axis

(please use the appropriate program function). Fit your data to the equations (18, 19) *or* (22, 23) and use them to determine f_G. *Note:* First, substitute $\omega_G = 1/RC$ into these equations and then simplify.

b) Determine the cutoff frequency by direct measurement using the AC millivoltmeter. To do this, use the circuit shown in Fig. 9, but with the high-pass *or* low-pass filter instead of the transformer. First, at a sufficiently low or high frequency (on the filter plateau, see Fig. 4), adjust the input voltage so that CH1 **and** CH2 display 0 dB. Now vary the frequency until CH2 reaches −3 dB; if necessary, the amplitude of the frequency generator must be manually adjusted so that CH1 reads exactly 0 dB . The frequency at which CH2 reaches −3 dB corresponds to f_G. Explain this using the definition of the cutoff frequency on page 4, converted to a level measurement using the definition of the decibel unit. Compare the directly measured f_G with the theoretical value for the cutoff frequency (based on independent measurements of R and C!) and perform an error analysis. Also, check the phase difference between input and output at f_G.

2. Now use the transformer provided to you, which has a 1:10 ratio between the numbers of turns of both coils.

a) Use LabVIEW to automatically record the transfer function of the transformer, see Fig. 8. Start at $f = 10$ Hz and measure up to a frequency above the system's resonance frequency. Select a suitable input amplitude! In regions where there are large changes, you need to record a sufficient number of data points. Determine the cutoff frequencies $f_{G,o}$ and $f_{G,u}$ of the transformer. What is the operating range of this transformer (specific frequency limits)? Compare the measured amplitude transfer ratio $|k_{II}|$ in the plateau region with the ratio between the numbers of turns of both coils. Also, determine the resonance frequency f_R and the width of the resonance peak Δf, and use these values to calculate the quality factor Q of the resonance. What transmission ratio is achieved at resonance? What does it depend on?

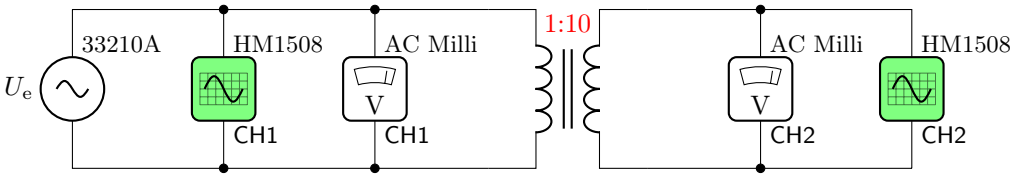


Figure 9: Circuit for manually measuring the transfer function of a transformer.

b) Now manually check the amplitude and phase response, see Fig. 9. Use three different frequencies: one in the plateau region, one at the resonance frequency, and one approximately 10 kHz above the resonance frequency. Compare these manually recorded values with the automatic LabVIEW measurement in the same diagram and check for consistency.

c) Repeat task b) for $f = 1$ Hz. Focus here on describing the waveforms CH1 and CH2 as displayed on an oscilloscope in digital mode. What particular difficulties arise when measuring the transfer function of a transformer at such low frequencies, and why?

d) Determine the resonance frequency f_R of the transformer using Lissajous figures on the oscilloscope in "XY" mode, see Fig. 2. What is the phase at f_R?