350 – Electronic Measurement Methods

Acquired core competencies:

1. You develop strategies for collecting data over wide measurement ranges.
2. You critically question the applicability of various measuring instruments and independently select suitable instruments appropriate for future measurement tasks.
3. You check measured values for plausibility in order to identify systematic sources of error in the measurement setup and, if possible, correct them.

1 Tasks

1. Investigation of the frequency response of various voltage meters.
2. Measurement of various voltage signals (sine wave, triangle wave, square wave) using an oscilloscope and comparison of the values with the reading on an AC millivoltmeter.
3. Determination of the room temperature and the boiling point of nitrogen using a Pt100 resistance sensor in both 2-wire and 4-wire configurations.

2 Fundamentals

Keywords: DC and AC voltage, Ohm's law, internal resistance of ammeters and voltmeters, oscilloscope, root mean square (RMS) value, average rectified value (ARV), moving-coil measuring instrument, temperature measurement using Pt100, 2-wire and 4-wire measurement

2.1 Basic principles of electrical circuits

If a voltage U is applied to an ohmic conductor, a current I flows through it. (*Please note:* In contrast to Anglo-American conventions we generally use U to denote voltages.) The magnitude of the current is determined by the conductor's ohmic resistance R. Ohm's law states:

$$R = \frac{U}{I} \tag{1}$$

In unbranched and branched circuits (see Fig. 1), Kirchhoff's circuit laws apply. As a direct consequence of the continuity equation, we obtain:

$$\textbf{junction rule:} \qquad \sum_n I_n = 0 \qquad \text{(for a given junction)} \tag{2}$$

In equation (2), all currents flowing into a given node are counted as positive, and all currents flowing out of it are counted as negative. In other words: The sum of the currents flowing into a node equals the sum of the currents flowing out of it. Furthermore, we can conclude from Faraday's law of induction:

$$\textbf{loop rule:} \qquad \sum_m U_m = 0 \qquad \text{(for a given loop)} \tag{3}$$

Here, too, the voltages are signed quantities: A positive sign is chosen if the direction of current or voltage at an electrical component is the same as the direction of the loop (otherwise, a

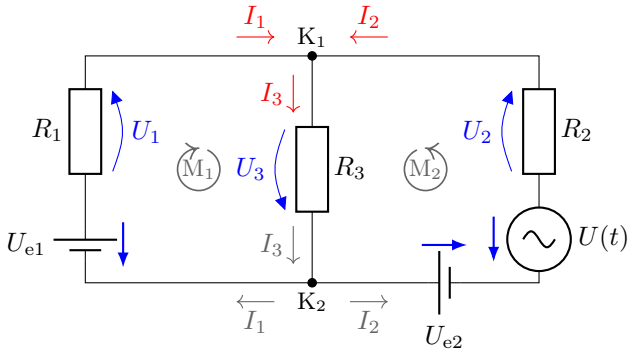


Figure 1: A branched circuit with two loops, each of which has been arbitrarily assigned a direction. From equation (2) it follows that: $I_1 + I_2 - I_3 = 0$. From equation (3) it follows that: $U_1 + U_3 - U_{e1} = 0$ for M_1 and $U_2 + U_3 + U_{e2} - U(t) = 0$ for M_2.

negative sign). For DC voltage sources, the applied voltage from the positive terminal to the negative terminal is defined as positive (see arrows at U_{e1} and U_{e2} in Fig. 1).

But how do you assign the direction of the loop "correctly"? Short answer: It doesn't really matter. Slightly longer answer: [i] Number all loops in the circuit. [ii] For each loop, choose a direction. [iii] Draw voltage drops of the *DC voltage sources* ($+ \longrightarrow -$). For *AC voltage sources* this arrow needs to be defined. [iv] Draw the current directions and voltage drops across the *loads* according to the direction of the loop. Leave any branches that have already been defined (e.g., in the middle of the circuit in Fig. 1) *unchanged*.

An important application is the so-called voltage divider shown in Fig. 2. It consists of a series connection of (in the simplest case, two) resistors. Since this is a simple, unbranched circuit, the current flowing into each component must be equal to the current flowing out of it, i.e., the same current $I_1 = I_2 = I$ flows through all components. The partial voltages across the resistors are related according to equation (1) as follows:

$$\boxed{\frac{U_1}{U_2} = \frac{R_1}{R_2}} \tag{4}$$

The voltage divider rule (4) states that the higher voltage drops across the larger resistor. In many cases, this form is impractical. To calculate the partial voltage U_1, one would need to know the (often unknown) partial voltage U_2. Often, however, only the total voltage U_e and the values of the resistances are known. The total voltage is the sum of the partial voltages ($U_e = U_1 + U_2$). After some rearrangements, we obtain:

$$\boxed{U_1 = U_e \cdot \frac{R_1}{R_1 + R_2}} \tag{5}$$

Equation (5) is often needed and should be memorized: The voltage across component X is equal to the total voltage multiplied by the resistance of X, divided by the total resistance.

2.2 Measurement of current and voltage

Electric currents and voltages can be measured using moving-coil meters. An electric current flows through the device to generate a magnetic field. This field is used to deflect a pointer, whereby the deflection is proportional to the current flowing through the meter.

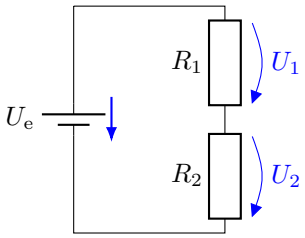
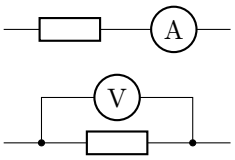


Figure 2: A voltage divider with two resistors, R_1 and R_2. The same current I flows through both resistors. From equation (3) it follows: $U_1 + U_2 - U_e = 0$, i.e., the partial voltages U_1 and U_2 add up to the total voltage U_e supplied by the voltage source.

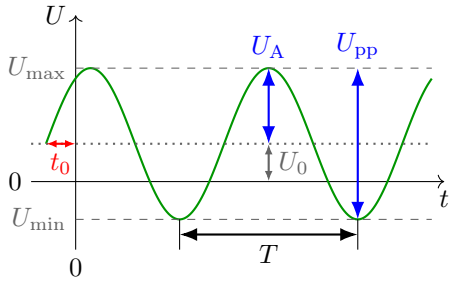
Currents are measured in series with the component in question, voltages are measured in parallel with the component. A practical measuring instrument has an internal resistance, because a voltage drop always occurs across the instrument when a current flows (or, conversely, a current flows when a voltage is applied; see the moving-coil meter, in which the current generates a magnetic field).



A practical **ammeter** has a very **low internal resistance** $R > 0$.
A practical **voltmeter** has a very **high internal resistance** $R < \infty$.

2.3 AC voltage and RMS value

Alternating current and alternating voltage make it possible to transport large amounts of energy over long distances with minimal loss (through transformation) as well as to transmit signals (information is encoded in the amplitude or frequency). Figure 3 specifies the key parameters of an alternating voltage. These are primarily the amplitude U_A, the phase φ_0, the periodic time T, and the frequency $f = 1/T$. The abbreviations DC (direct current) and AC (alternating current) are used for both currents and voltages.



U_0 ... offset voltage ("DC part")
U_A ... amplitude ("AC part")
U_{pp} ... peak-to-peak voltage
T ... periodic time
f ... frequency ($f = 1/T$)
ω ... angular frequency ($\omega = 2\pi f = 2\pi/T$)
φ_0 ... initial phase ($\varphi_0 = \omega \cdot t_0$)

Figure 3: Key parameters of an AC voltage $U(t) = U_0 + U_A \cdot \sin(\omega t + \varphi_0)$. For the sinusoidal component (after subtracting the offset), the following applies: $U_A = U_{pp}/2$.

The root mean square value U_{eff} of an alternating voltage is defined as the equivalent DC voltage that, when applied to an ohmic resistor R, delivers the same power $P = U^2/R$ as the alternating voltage *on average over time*. This results in:

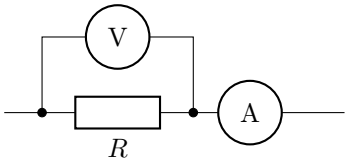
$$U_{eff} = \sqrt{\frac{1}{T} \cdot \int_0^T U^2(t)\,\mathrm{d}t} \tag{6}$$

with the periodic time T and the time trace of the alternating voltage $U(t)$. For a sinusoidal alternating voltage $U(t) = U_A \cdot \sin(\omega t)$, the root-mean-square value is $U_{eff} = U_A/\sqrt{2}$.

2.4 Determining electrical resistance by measuring current and voltage simultaneously

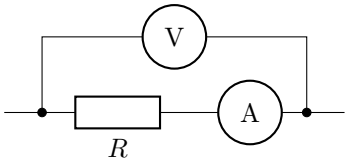
In order to experimentally determine the ohmic resistance R according to equation (1), both the voltage across the component and the current through the component must be known simultaneously. This often leads to measurement problems, as described below.

2.4.1 Voltage-based measurement



One method for determining U and I across / through an electrical component is shown on the right. Here, the voltage is measured directly across the component, and the current is measured in the common wire, which ultimately leads to the voltage source. While the measured U is the true voltage across the component, the measured I represents the sum of the current through the component (which is the one being measured) and the current through the voltmeter (note the internal resistance of practical measuring instruments!). This inevitably results in a systematic measurement error. This error becomes particularly apparent when the current through the voltmeter is on the same order of magnitude as the current through the component under test. Therefore, voltage-based measurements are unsuitable for high-resistance components.

2.4.2 Current-based measurement



In contrast, the circuit shown on the right illustrates the current-based measurement. In this case, the current flowing through the component is measured directly, while the voltage is measured across the component and the ammeter. This circuit is unsuitable whenever the voltage drop across the ammeter is on the same order of magnitude as the voltage drop across the component. Therefore, current-based measurements are unsuitable for low-resistance components.

2.4.3 2-wire and 4-wire measurements

In many experimental setups, it is necessary to determine the electrical resistance of a component with long wires. Examples include temperature measurements at very low temperatures. In many cases, several meters of cable are required here, since the distance between the hot and cold ends must be very large due to the insulation of the experimental setup. Other examples include sensors in hazardous environments (e.g., radiation-protected areas) that must be read out via long cables.

In such cases, the inherent resistance of the wires becomes a factor in the precise determination of the sensor's electrical resistance. Figure 4 illustrates a typical measurement problem. A temperature sensor is to be operated in an inaccessible location and read out via long cables in an accessible area. As a result, current and voltage measuring instruments are located far away from the actual measurement object.

For the purposes of further analysis, we will assume, for simplicity, a medium-sized resistor R, so that the problem of voltage-based and current-based measurements can be neglected. Under this assumption, the ammeter measures the true current through the sensor resistor R (series connection). However, the voltage drop measured by the voltmeter corresponds to the voltage across the sensor resistor and the connecting wires: $U_{\mathrm{meas}} = U_{R_Z} + U_R$. If we now use the measured voltage U_{meas} and the measured current I_{meas} to calculate the resistance R_{meas}, the result is too high:

$$R_{\mathrm{meas}} = \frac{U_{\mathrm{meas}}}{I_{\mathrm{meas}}} = \frac{U_{R_Z} + U_R}{I_{\mathrm{meas}}} > \frac{U_R}{I_{\mathrm{meas}}} = R \tag{7}$$

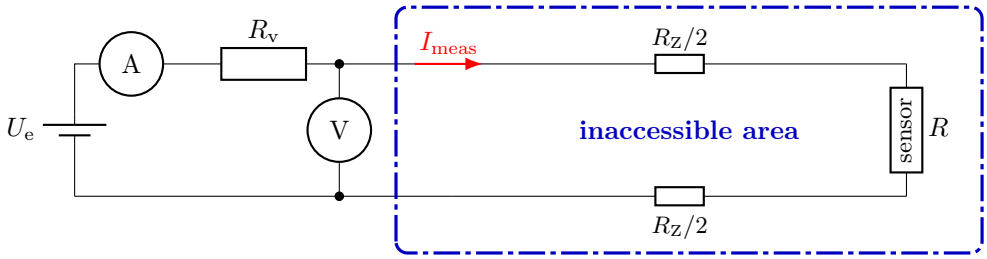


Figure 4: Read-out of a sensor using a 2-wire measurement. For clarity, the total resistance of the wires to the sensor is assumed to be R_Z.

This systematic measurement error becomes all the more serious the greater the wire resistance R_Z is relative to the resistance R.

To determine the resistance R as accurately as possible, one could measure the resistance R_Z of the wires separately (by short-circuiting the sensor) and later subtract it from the measured total resistance. In many cases, however, this is not possible because the resistance of the wires may change over time in unpredictable ways (e.g., due to temperature fluctuations). In such cases, a 4-wire measurement is used to eliminate the error caused by the wire resistance.

Figure 5 shows the 4-wire configuration. In this setup, an additional pair of wires is routed from the sensor to the external voltmeter. This pair of wires also has a total resistance of R_Z (assuming the same geometries and materials).

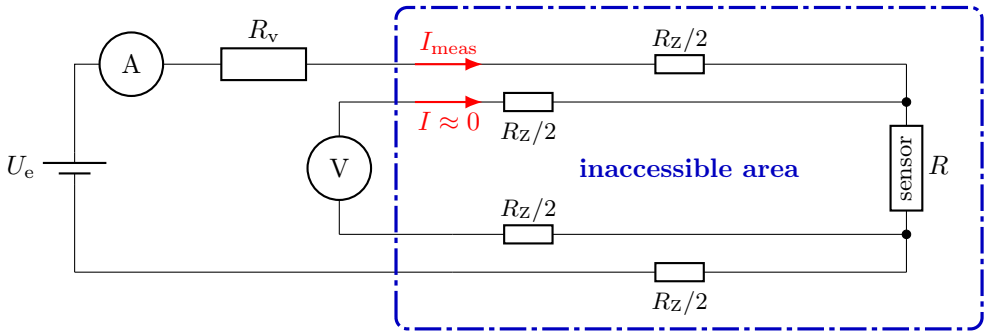


Figure 5: Read-out of a sensor using a 4-wire measurement. Compared to a simple 2-wire measurement, the voltmeter is connected to the sensor using an additional pair of wires (with a total resistance of R_Z).

Due to the high internal resistance of the voltmeter (typically $> 1\,\mathrm{M\Omega}$), virtually no current flows in this circuit ($I \approx 0$). It follows that no voltage drop occurs across the wires to the voltmeter in this circuit. Consequently, the voltmeter displays the true voltage $U_R = R \cdot I_{\mathrm{meas}}$ at the sensor. The current I_{meas} supplied by the voltage source flows only through the outer circuit – that is, also through the sensor R – and is correctly measured by the ammeter. Hence, by separating the wires for current and voltage measurements, the influence of the wire resistance is eliminated. Even if the value of R_Z changes during the measurement, this has no effect on the determination of the sensor resistance R.

The 4-wire configuration is a voltage-based measurement and is therefore suitable for resistances that are significantly smaller than the internal resistance of the voltmeter. Since this internal resistance is very large ($> 1\,\mathrm{M\Omega}$), this measurement method is applicable to nearly all practical problems. In particular, the 4-wire configuration is mandatory for very small resistances (for example, $\ll 1\,\Omega$), since in this case even highly conductive copper wires would produce noticeable measurement errors in a 2-wire configuration.

2.5 Resistive temperature measurement

To measure a physical quantity electrically, one must use a corresponding effect that links that physical quantity to an electrical quantity. The electrical resistance of many conductive materials depends on temperature. For various types of conductors, different temperature dependencies are observed, see Fig. 6(a).

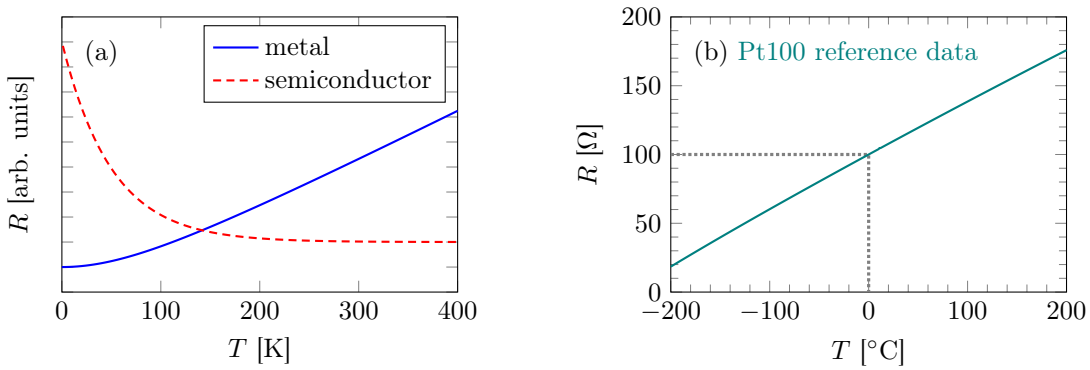


Figure 6: (a) Typical qualitative dependence of a sample's electrical resistance on temperature (schematic). The axis for R is arbitrarily scaled. (b) Depiction of the standardized $R(T)$ values of a Pt100 resistor, compare Table A1.

In metallic conductors, electric current is carried by almost freely moving electrons (electron gas model). Their flow is impeded by lattice vibrations (a.k.a. phonons) or collisions with impurity atoms. If the temperature of such a sample is lowered, the number and amplitude of the lattice vibrations decrease – the flow of current is less impeded, and the electrical resistance consequently decreases. The contribution of impurity atoms to the electrical resistance, on the other hand, is independent of temperature. As a result, a constant residual resistance remains, especially at very low temperatures (where lattice vibrations are negligible).

2.6 Measurements over a wide frequency range

Often, a meaningful representation of a physical quantity requires varying the corresponding variables over a wide range, usually spanning several orders of magnitude. Figure 7 illustrates this using the phase transfer function φ (as a function of frequency f) of an RC low-pass filter. When plotting the theoretically expected function $\varphi(f)$, it is noticeable that the curve (——) is much easier to discern with a *logarithmic* scale on the frequency axis than with a *linear* one.

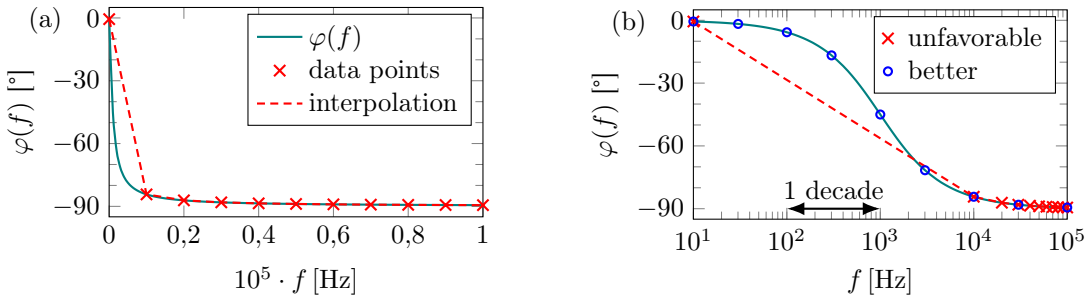


Figure 7: Two representations of the *exactly* identical function —— plotted (a) on a *linearly* scaled and (b) on a *logarithmically* scaled frequency axis. The data points × in (a) and (b) are at identical values! The data points ∘ in (b) are distributed such that two values are always recorded per decade, i.e., they are equidistant on the *logarithmically* scaled frequency axis.

If one wishes to measure the curve in a practical application, this is, of course, only possible with a finite number of data points. This raises the question of which values are most appropriate as sampling points. If one selects the values to be equidistantly distributed along the *linear* frequency axis (e.g., always at intervals of 10^4 Hz, marked by ×), it becomes apparent that the data points in the logarithmic plot shown in Fig. 7(b) are heavily concentrated in the right-hand region, while far too few data points are recorded along the rest of the curve. Therefore, it is much better to choose the sampling points such that they are equidistantly distributed along the *logarithmic* frequency axis, e.g., the symbols ∘ in Fig. 7(b).

☞ As you prepare for the experiment in advance, think about how to select the frequencies so that, on a *logarithmic* frequency axis, you always obtain exactly 4 equidistant sampling points per decade. Calculate these values rounded to 2 significant decimal places, compile them as a table in a data file, and prepare 3 additional empty columns for the experiment.

3 Experimental procedure

1. Connect the oscilloscope, the AC millivoltmeter, and the digital multimeter (DMM) in parallel to the output of the Agilent 33210A frequency generator (Fig. 8). Vary the frequency between 10 Hz and 10 MHz. *It is best to start at* 100 Hz *and initially vary the frequency by a factor of 10 in each step, first to lower frequencies and then to higher ones, to get an overview.* Record the voltage values displayed on all measuring instruments at appropriate frequency intervals (equidistant on a *logarithmic* frequency axis!). Plot the results in a diagram with a logarithmic frequency axis and discuss them in terms of the suitability of the various measuring instruments.

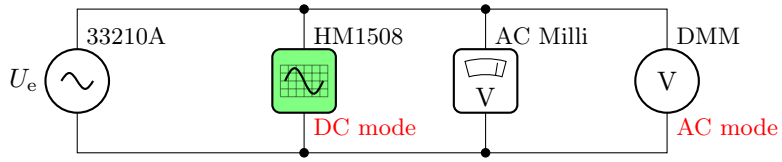


Figure 8: Comparison of the frequency response of different voltmeters. Set the input on the digital multimeter to AC mode (so the DMM measures only the AC component). Set the input on the oscilloscope to DC mode (the oscilloscope will then measure both the DC *and* AC parts of the signal). If you set the oscilloscope input to AC mode, low frequencies are filtered out by an internal high-pass filter (unlike with a DMM). This would systematically distort the measurement of low-frequency signals.

2. Repeat task 1 for $f = 1$ Hz. Focus here on describing the behavior of the three measuring instruments used. What specific difficulties arise when measuring AC voltages at such low frequencies, and why? Which of the three measuring devices can you use to obtain the most reliable reading?

   ⓘ Use the horizontal cursor lines in the analog mode of the Hameg HM1508 oscilloscope. Also, try out the digital mode of the HM1508.

3. Now apply an input signal of $f = 100$ Hz with an amplitude of $U_e = 1$ V to the circuit shown in Fig. 8. Read the displayed values for a sinusoidal, a triangular, and a square-wave voltage. How are the displayed values related to one another?

4. Set up a circuit for temperature measurements using a Pt100 resistor (see Fig. 9). To simulate a realistic application with long wires, the Pt100 sensor is mounted at the lower end of a measuring rod, which will be provided to you as a ready-to-use assembly. The wires are accessible at the upper end via two red and two black terminals.

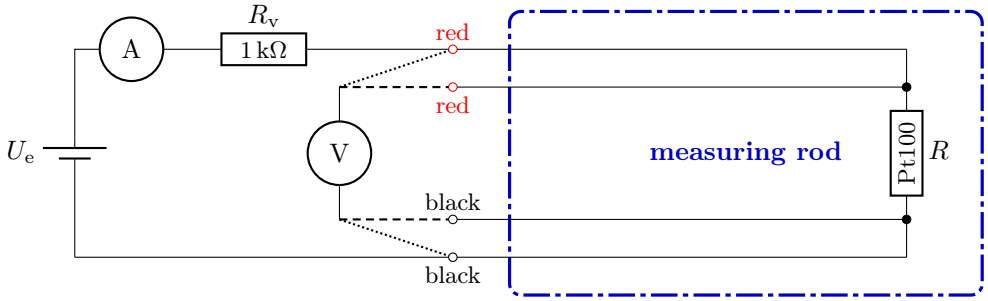


Figure 9: Circuit for temperature measurements using a Pt100 as a sensor. Depending on where the voltmeter is connected, either 2-wire (·····) or 4-wire (----) configurations can be realized. Make sure to include the series resistance $R_v = 1\,\mathrm{k\Omega}$.

a) Begin your experiments at room temperature. Determine the electrical resistance of the Pt100 sensor by measuring U and I simultaneously. Use $R_v = 1\,\mathrm{k\Omega}$ as a series resistor for this measurement. Vary U_e to set the current on the ammeter to approx. 1 mA. Perform the measurements using both 2-wire (Fig. 9 with ·····) and 4-wire (Fig. 9 with ----) configurations. Calculate the resistance R and, subsequently, the temperature T based on the 👉 Pt100 data set (also compiled in Table A1). Compare this value with the direct reading on a thermometer in the Electronics lab.

b) Measure the resistance of the Pt100 sensor again using the 4-wire configuration. Record values for R over a period of 5 min, increasing the measurement current to 15 mA **at the start of your data acquisition, *not* earlier!** Discuss the results.

c) Repeat the measurements from part a) with the following difference: Start **first** by measuring the resistance, and **then** immerse the measuring rod in liquid nitrogen. Observe the time trace $R(t)$ during the cooling process and record your measurements until they remain stable.

⚠ Safety advice for handling liquid nitrogen (−196 °C): Be sure to wear safety goggles! Always use a fitting base to secure the thermos jug!

d) Perform an error analysis for the temperature measurement. Note that the function $R(T)$ of the Pt100 is in good approximation linear within the relevant temperature range, see Fig. 6(b). How accurately can you determine the boiling point of nitrogen? Discuss the advantages and disadvantages of the 4-wire measurement method.

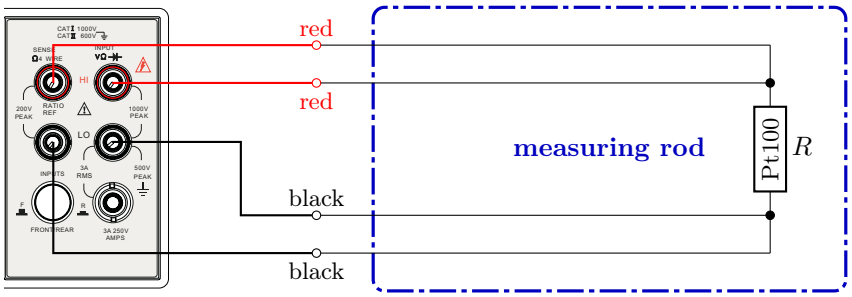


Figure 10: Temperature measurement using a Pt100 sensor and a Keithley 2100.

e) Use the Keithley 2100 precision measuring instrument (connection scheme in Fig. 10) to determine the ohmic resistance in 2-wire and 4-wire configurations. To do this, use the Ω2 and SHIFT + Ω4 keys, respectively. Compare the results obtained.

f) Use the Keithley 2100 precision measuring instrument (Fig. 10) for a direct temperature measurement in 4-wire configuration (using the TEMP key).

g) **Bonus:** Figure out where in Fig. 7 the 220 V power frequency should be plotted.

Appendix

Figure A1 briefly describes how to operate the GW Instek GVT-427B analog AC millivoltmeter. It also uses an example to explain the importance of selecting the correct measurement range. The following basic rule applies to all measuring instruments (including digital ones!):

> Always select the **lowest measurement range** at which the value is displayed without exceeding the range. In addition, all selected measurement ranges must be documented!

Reading a voltage of 100 mV in the measurement range of 300 mV results in an unnecessarily high uncertainty (instrument error + reading error) ⇒ it is like "driving with a foot on the brake". In this case, selecting the next lower measurement range, 100 mV, is clearly better.



Figure A1: Voltage measurements using the GW Instek GVT-427B analog AC millivoltmeter. An *identical* voltage was applied in parallel to both inputs. The knobs have a notch that points to the selected measurement range, highlighted here by green arrows. For measurement ranges beginning with a **1**, the values are read from the top black scale ⇒ for CH1, the corresponding black pointer reads 1.0, which consequently corresponds to 100 mV. For measurement ranges beginning with a **3**, the values are read on the second black scale from the top ⇒ for CH2, the corresponding red pointer also reads 1.0 here, which corresponds to 100 mV. The manufacturer's specification for measurement uncertainty is ±3 % **of the measurement range**, i.e., in our example, ±3 mV for CH1 and ±9 mV for CH2. The subjective uncertainty of the reading ("reading error") must be added, especially when a pointer is not positioned exactly over a scale mark. This relative uncertainty of the reading is also lowest near the pointer's full deflection. Here, the black pointer is optimal.

Error analysis (example)

You want to determine the resistance of a Pt100 sensor and later use that value to calculate the temperature. You measure a DC current of 1,000 mA using the Metrix MX22 in the 4 mA measurement range. In this case, the uncertainty is composed as follows: 1,0 % of the reading plus 3 digits (manufacturer's specifications), i.e., $1{,}0\,\% \cdot 1{,}000\,\text{mA} + 3 \cdot 0{,}001\,\text{mA} = 0{,}013\,\text{mA}$. Result: $I = (1{,}000 \pm 0{,}013)\,\text{mA}$. You also measure a DC voltage of 90,1234 mV using the Keithley 2100 in the 100 mV measurement range. In this case, the uncertainty is composed as follows: 0,0055 % of the reading plus 0,0040 % of the measurement range (manufacturer's specifications), that is, $0{,}0055\,\% \cdot 90{,}1234\,\text{mV} + 0{,}0040\,\% \cdot 100\,\text{mV} = 0{,}0090\,\text{mV}$. Result: $U = (90{,}123 \pm 0{,}009)\,\text{mV}$. This gives $R = U/I = 90{,}123\,\Omega$. Because powers of one appear here, the formula for the total relative error is: $|\Delta R/R| = |\Delta U/U| + |\Delta I/I| = |0{,}009\,\text{mV}/90{,}123\,\text{mV}| + |0{,}013\,\text{mA}/1{,}000\,\text{mA}| = 0{,}0001 + 0{,}013 \approx 1{,}3\,\%$. Please never forget the absolute value bars! Therefore, $|\Delta R| = 1{,}3\,\% \cdot 90{,}123\,\Omega = 1{,}2\,\Omega$. Thus, the final result is: $\underline{\underline{R = (90{,}1 \pm 1{,}2)\,\Omega}}$. It is not meaningful to specify more decimal places here, since the uncertainty is greater than 1 Ω.

Table A1: Excerpt from the Pt100 resistance data sheet (according to DIN EN IEC 60751). Copying + pasting the values from this table works with most PDF viewers.

T [°C]	R [Ω]	T [°C]	R [Ω]	T [°C]	R [Ω]	T [°C]	R [Ω]	T [°C]	R [Ω]
-200	18,493	-125	50,055	-50	80,307	25	109,734	100	138,500
-199	18,926	-124	50,466	-49	80,704	26	110,122	101	138,879
-198	19,358	-123	50,876	-48	81,101	27	110,509	102	139,258
-197	19,790	-122	51,286	-47	81,498	28	110,897	103	139,637
-196	20,221	-121	51,696	-46	81,894	29	111,284	104	140,016
-195	20,653	-120	52,106	-45	82,291	30	111,672	105	140,395
-194	21,083	-119	52,515	-44	82,687	31	112,059	106	140,773
-193	21,514	-118	52,924	-43	83,083	32	112,446	107	141,152
-192	21,944	-117	53,333	-42	83,479	33	112,833	108	141,530
-191	22,374	-116	53,742	-41	83,875	34	113,220	109	141,908
-190	22,803	-115	54,151	-40	84,271	35	113,607	110	142,286
-189	23,232	-114	54,559	-39	84,667	36	113,994	111	142,664
-188	23,661	-113	54,967	-38	85,063	37	114,380	112	143,042
-187	24,089	-112	55,375	-37	85,458	38	114,767	113	143,420
-186	24,517	-111	55,783	-36	85,853	39	115,153	114	143,797
-185	24,945	-110	56,190	-35	86,248	40	115,539	115	144,175
-184	25,372	-109	56,598	-34	86,643	41	115,925	116	144,552
-183	25,799	-108	57,005	-33	87,038	42	116,311	117	144,930
-182	26,226	-107	57,412	-32	87,433	43	116,697	118	145,307
-181	26,652	-106	57,818	-31	87,828	44	117,083	119	145,684
-180	27,078	-105	58,225	-30	88,222	45	117,469	120	146,061
-179	27,504	-104	58,631	-29	88,617	46	117,854	121	146,438
-178	27,929	-103	59,037	-28	89,011	47	118,240	122	146,814
-177	28,354	-102	59,443	-27	89,405	48	118,625	123	147,191
-176	28,779	-101	59,849	-26	89,799	49	119,010	124	147,567
-175	29,203	-100	60,254	-25	90,193	50	119,395	125	147,944
-174	29,627	-99	60,659	-24	90,587	51	119,780	126	148,320
-173	30,051	-98	61,065	-23	90,980	52	120,165	127	148,696
-172	30,474	-97	61,469	-22	91,374	53	120,550	128	149,072
-171	30,897	-96	61,874	-21	91,767	54	120,934	129	149,448
-170	31,320	-95	62,279	-20	92,160	55	121,319	130	149,824
-169	31,742	-94	62,683	-19	92,553	56	121,703	131	150,199
-168	32,165	-93	63,087	-18	92,946	57	122,087	132	150,575
-167	32,587	-92	63,491	-17	93,339	58	122,471	133	150,950
-166	33,008	-91	63,895	-16	93,732	59	122,855	134	151,326
-165	33,429	-90	64,299	-15	94,125	60	123,239	135	151,701
-164	33,850	-89	64,702	-14	94,517	61	123,623	136	152,076
-163	34,271	-88	65,105	-13	94,910	62	124,007	137	152,451
-162	34,691	-87	65,508	-12	95,302	63	124,390	138	152,826
-161	35,111	-86	65,911	-11	95,694	64	124,774	139	153,200
-160	35,531	-85	66,314	-10	96,086	65	125,157	140	153,575
-159	35,951	-84	66,717	-9	96,478	66	125,540	141	153,950
-158	36,370	-83	67,119	-8	96,870	67	125,923	142	154,324
-157	36,789	-82	67,521	-7	97,262	68	126,306	143	154,698
-156	37,208	-81	67,923	-6	97,653	69	126,689	144	155,072
-155	37,626	-80	68,325	-5	98,045	70	127,072	145	155,446
-154	38,044	-79	68,727	-4	98,436	71	127,454	146	155,820
-153	38,462	-78	69,128	-3	98,827	72	127,837	147	156,194
-152	38,879	-77	69,530	-2	99,218	73	128,219	148	156,568
-151	39,297	-76	69,931	-1	99,609	74	128,602	149	156,941
-150	39,714	-75	70,332	0	100,000	75	128,984	150	157,315
-149	40,130	-74	70,733	1	100,391	76	129,366	151	157,688
-148	40,547	-73	71,134	2	100,781	77	129,748	152	158,061
-147	40,963	-72	71,534	3	101,172	78	130,130	153	158,435
-146	41,379	-71	71,934	4	101,562	79	130,511	154	158,808
-145	41,795	-70	72,335	5	101,953	80	130,893	155	159,180
-144	42,210	-69	72,735	6	102,343	81	131,274	156	159,553
-143	42,625	-68	73,135	7	102,733	82	131,656	157	159,926
-142	43,040	-67	73,534	8	103,123	83	132,037	158	160,298
-141	43,455	-66	73,934	9	103,513	84	132,418	159	160,671
-140	43,869	-65	74,333	10	103,902	85	132,799	160	161,043
-139	44,283	-64	74,733	11	104,681	86	133,180	161	161,415
-138	44,697	-63	75,132	12	104,681	87	133,561	162	161,787
-137	45,111	-62	75,531	13	105,071	88	133,941	163	162,159
-136	45,524	-61	75,930	14	105,460	89	134,322	164	162,531
-135	45,937	-60	76,328	15	105,849	90	134,702	165	162,903
-134	46,350	-59	76,727	16	106,238	91	135,083	166	163,274
-133	46,763	-58	77,125	17	106,627	92	135,463	167	163,646
-132	47,175	-57	77,523	18	107,016	93	135,843	168	164,017
-131	47,587	-56	77,921	19	107,404	94	136,223	169	164,388
-130	47,999	-55	78,319	20	107,793	95	136,603	170	164,760
-129	48,411	-54	78,717	21	108,181	96	136,982	171	165,131
-128	48,822	-53	79,115	22	108,570	97	137,362	172	165,501
-127	49,234	-52	79,512	23	108,958	98	137,741	173	165,872
-126	49,645	-51	79,910	24	109,346	99	138,121	174	166,243